Ojlerov algoritam

$$dy/dx = g(x,y)$$

g(x,y) je brzina promene funkcije y u tački x
u prvoj aproksimaciji je konstantna na intervalu $(x_0 - x_1)$

$$x = x_0 \quad y(x_0) = y_0$$

$$x_1 = x_0 + \Delta x$$

$$y_1 = y(x_0) + \Delta y \sim y(x_0) + g(x_0,y_0)\,\Delta x$$

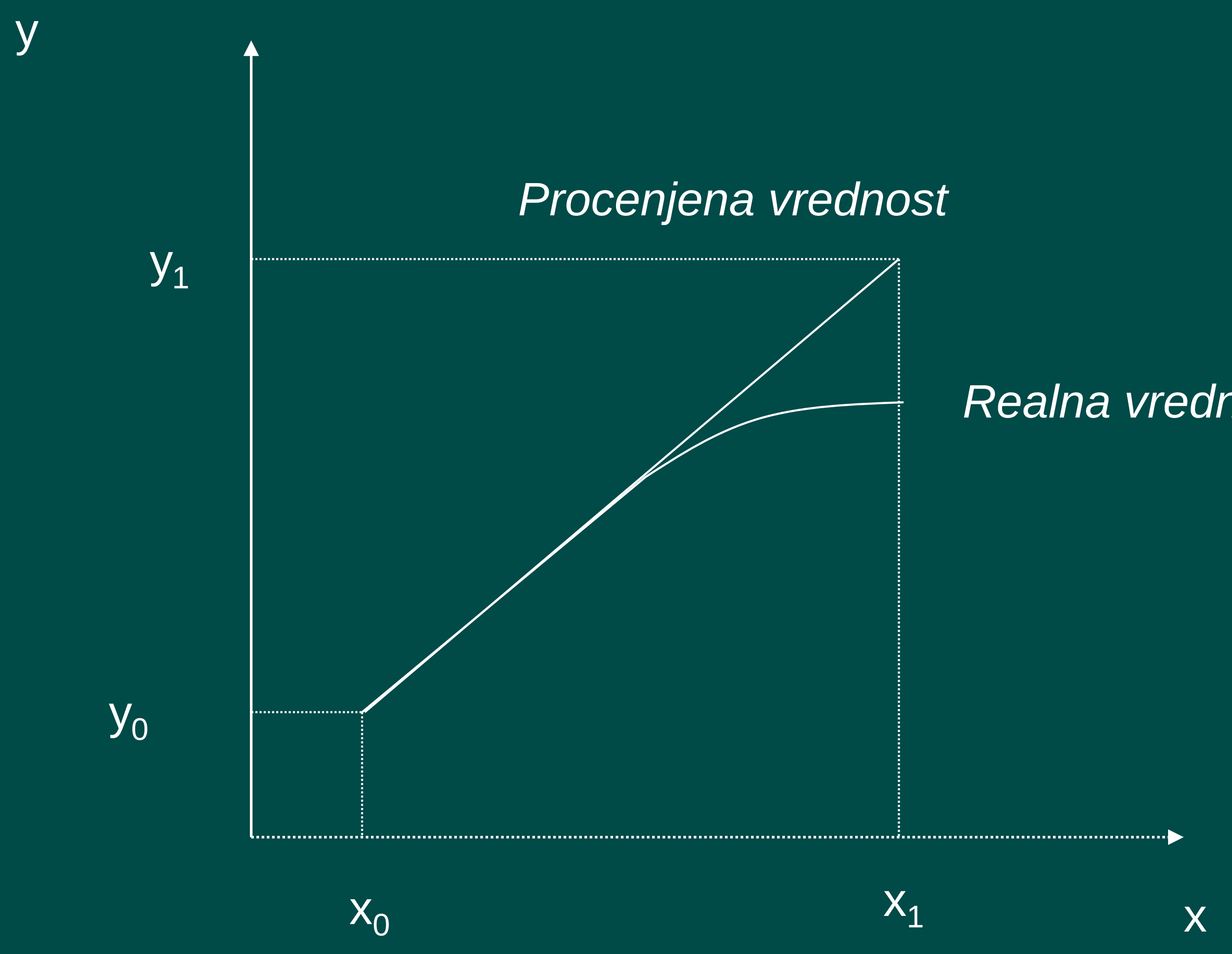
$$x_2 = x_1 + \Delta x$$

$$y_2 = y(x_1 + \Delta x) \sim y(x_1) + g(x_1, y_1)\, \Delta x$$

$$x_n = x_0 + n\Delta x$$

$$y_n = y_{n-1} + g(x_{n-1}, y_{n-1})\, \Delta x \quad (n = 0, 1, 2, \ldots)$$

Grafički prikaz Ojlerovog algoritma



Jednostavni primer

$$dy/dx = 2x$$

$$x = x_0 = 1 \quad y(x_0) = y_0 = 1$$

Tražimo približnu vrednost funkcije u tački $x = 2$.

$$g(x_0) = 2x_0 = 2, \ \Delta x = 0.1, \ n = (2-1)/0.1 = 10$$

$$y_1 = y(x_0) + g(x_0)\,\Delta x = 1 + 2 \cdot 0.1 = 1.2$$

$$x_1 = x_0 + \Delta x = 1 + 0.1 = 1.1$$

$$g(x_1) = 2x_1 = 2.2$$

$$y_2 = y(x_1) + g(x_1)\,\Delta x = 1.2 + 2.2 \cdot 0.1 = 1.42$$

Za $x = 2$ dobijamo $y = 3.90$, a tačno rešenje daje $y = x^2 = 4$. Odstupanje je (4-3.90)/4 = 0,025 = = 2,5 %.

Algoritam za metod Ojlera

1. Biraju se početni uslovi, veličina koraka i broj koraka.
2. Određuje se koeficijent pravca u početnoj tački intervala.
3. Izračunava se vrednost *y* u krajnjoj tački intervala i zapisuje rezultat.
4. Koraci 2. i 3. Ponavljaju se potreban broj puta.

Program example

```
# include <stdio.h>
# include <math.h>
int main ()
{

        FILE *fp1;
        fp1=fopen("example.data","w");
        double y,x,xmax,dx,slope,change;

        x=1;
        y=1;
        xmax=2;
        dx=0.1;
```

nastavak

```
while (x  <= xmax)
{

    slope=2*x;
    change=slope*dx;
    y=y+change;
    x=x+dx;
            fprintf(fp1,"%10.3f,%10.3f,\n",x,y);

}
fclose(fp1);
}
```

Njutnov zakon hladjenja

$$\mathrm{d}T/\mathrm{d}t = -r\,(T-T_s)$$

T-temperatura tela
T_s-temperatura okoline
r-konstanta hladjenja

Program cool

```
# include <stdio.h>
# include <math.h>

main ()
{

        FILE *fp1;
        fp1=fopen("cool.data","w");
        double
Temp,t,tmax,dt,slope,change,r,Temps;

        t=0;
        Temp=82.3;
        Temps=17.;
        r=0.1;
        tmax=30;
        dt=0.1;
```

nastavak

```
while (t  <= tmax)
{

    slope=-r*(Temp-Temps);
    change=slope*dt;
    Temp=Temp+change;
    t=t+dt;
            fprintf(fp1,"%10.3f,%10.3f,\n",t,Temp);

}
fclose(fp1);
}
```

Tabela eksperimentalnih rezultata

Vreme (min)	Temperatura (C)	Vreme (min)	Temperatura (C)
0.	82.3	16.	58.1
2.	78.5	18.	56.1
4.	74.3	20.	54.3
6.	70.7	22.	52.8
8.	67.6	24.	51.2
10.	65.0	26.	49.9
12.	62.5	28.	48.6
14.	60.1	30.	47.2

Tačnost i stabilnost Ojlerove metode

$$\frac{dT}{dt} = -r(T - T_s)$$

$$\int \frac{dT}{T - T_s} = -\int r\,dt$$

$$\ln(T - T_s) = -rt + \ln c$$

$$\ln \frac{T - T_s}{c} = -rt$$

$$T - T_s = ce^{-rt}$$

$$t = t_0 \Rightarrow c = T_0 - T_s$$

$$T - T_s = (T_0 - T_s)e^{-rt}$$

$$T(t) = T_s - (T_s - T_0)e^{-rt}$$

$$T(t = 0) = T_s - (T_s - T_0) = T_0$$

$$T(t \to \infty) = T_s$$

Zadatak

$$R\frac{dQ}{dt} = V - \frac{Q}{c}$$

$$t = 0 \Rightarrow Q = 0$$

$$R = 2000\,\Omega$$

$$c = 10^{-6}F$$

$$V = 10V$$